Essential Mathematics for the Twenty-first Century:
The Position of the National Council of Supervisors of Mathematics

Since *A Nation at Risk* was released five years ago, we have experienced a most serious and sustained national effort to improve education. That attention has been welcomed, and it requires our continued support.

Today, reformers continue their search for ways to effect the curricular changes needed. Evidence is mounting that principal among their major concerns is mathematics. The NCSM's "Essential Mathematics for the Twenty-first Century" was created for the purpose of contributing to current efforts to reform mathematics education.

This NCSM position statement is intended to complement the mathematics curriculum as developed in the NCTM's *Standards* and reflect that commitment to the evolving changes. The components are the basic mathematical competencies that represent those elements critical to the revision and expansion of mathematics programs. For the many critical audiences, mathematics educators, the broader educational community, and other decision makers, it is designed to be a concise proposal for mathematics curricular reform.

A successful response from the mathematics education community to the reform effort will require many levels and forms of support. "Essential Mathematics for the Twenty-first Century" is one facet developed from the perspective of school-based mathematics educators, whose specific assignments are curriculum and instruction. We hope that it will assist in the process of reaching a consensus for excellence in mathematics education that will meet the demands of the twenty-first century.

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Introduction

Students who enter kindergarten in 1988 can expect to graduate from high school in the year 2001. Yet these students who will graduate in the twenty-first century still frequently face a computation-dominated curriculum more suitable for the nineteenth century. To address this inconsistency, the National Council of Supervisors of Mathematics (NCSM) now updates its 1977 basic skills position statement to describe the essential mathematical competencies that citizens will need to begin adulthood in the next millenium. This position by NCSM is intended to complement and support positions on mathematics education by the National Council of Teachers of Mathematics and other professional groups.

Our technological world is changing at an ever-increasing rate, and our responsibilities in international affairs continue to increase. As the demands of society change, so do the essential competencies needed by individuals to live productively in that society. *All* students, including those of all races and both sexes, will need competence in essential areas of mathematics.

What Is Essential?

The NCSM views as "essential" those competencies that are necessary for the doors to employment and further education to remain open. Essential mathematics, as described in this paper, represents the mathematical competence students will need for responsible adulthood.

The students we educate today can expect to change jobs many times during their lifetimes. The jobs they hold will develop and change around them. Often, specific job skills will not transfer from one position to another. To prepare for mobility, students must develop a thorough understanding of mathematical concepts and principles; they must reason clearly and communicate effectively; they must recognize mathematical applications in the world around them; and they must approach mathematical problems with confidence. Individuals will need the fundamental skills that will enable them to apply their knowledge to new situations and to take control of their own lifelong learning.

Skill in whole-number computation is not an adequate indicator of mathematical achievement. Nor is it sufficient to develop skills apart from their applications or to memorize rules without understanding the concepts on which they are based. Students must understand mathematical principles; they must know when and how to use computation; and they must develop proficiency in problem solving and higher-order thinking.

The NCSM position statement of

This position statement has been endorsed by the National Council of Teachers of Mathematics.

1977 responded to the "back-to-basics" movement with its overly narrow conception of basic skills. Now, as we look to the future, we recognize that the use of calculators and computers and the application of statistical methods will continue to expand. Creative problem solving, precise reasoning, and effective communication will grow in importance. To function effectively in the next century, students will need proficiency in an enriched body of essential mathematics. The list that follows identifies twelve critical areas of mathematical competence for all students. It does not imply an instructional sequence or a priority among topics. In fact, the twelve essential mathematics areas are interrelated; competence in each area requires competence in other areas.

Twelve Components of Essential Mathematics

Problem solving

Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with nontext problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. Students should see alternate solutions to problems; they should experience problems with more than a single solution.

Communicating mathematical ideas

Students should learn the language and notation of mathematics. For example, they should understand place value and scientific notation. They should learn to receive mathematical ideas through listening, reading, and visualizing. They should be able to present mathematical ideas by speaking, writing, drawing pictures and graphs, and demonstrating with concrete models. They should be able to discuss mathematics and ask questions about mathematics.

Mathematical reasoning

Students should learn to make independent investigations of mathematical ideas. They should be able to identify and extend patterns and use experiences and observations to make conjectures (tentative conclusions). They should learn to use a counterexample to disprove a conjecture, and they should learn to use models, known facts, and logical arguments to validate a conjecture. They should be able to distinguish between valid and invalid arguments.

Applying mathematics to everyday situations

Students should be encouraged to take everyday situations, translate them into mathematical representations (graphs, tables, diagrams, or mathematical expressions), process the mathematics, and interpret the results in light of the initial situation. They should be able to solve ratio, proportion, percent, direct-variation, and inverse-variation problems. Not only should students see how mathematics is applied in the real world, but they should observe how mathematics grows from the world around them.

Alertness to the reasonableness of results

In solving problems, students should question the reasonableness of a solution or conjecture in relation to the original problem. Students must develop the number sense to determine if results of calculations are reasonable in relation to the original numbers and the operations used. With the increase in the use of calculating devices in society, this capability is more important than ever.

Estimation

Students should be able to carry out rapid approximate calculations through the use of mental arithmetic and a variety of computational estimation techniques. When computation is needed in a problem or consumer setting, an estimate can be used to check reasonableness, examine a conjecture, or make a decision. Students should acquire simple techniques for estimating such measurements as length, area, volume, and mass (weight). They should be able to decide when a particular result is precise enough for the purpose at hand.

Appropriate computational skills

Students should gain facility in using addition, subtraction, multiplication, and division with whole numbers and decimals. Today, long, complicated computations should be done with a calculator or computer. Knowledge of single-digit number facts is essential, and using mental arithmetic is a valuable skill. In learning to apply computation, students should have practice in choosing the appropriate computational method: mental arithmetic, paper-pencil algorithm, or calculating device. Moreover, everyday situations arise that demand recognition of, and simple computation with, common fractions. In addition, the ability to recognize, use, and estimate with percents must also be developed and maintained.

Algebraic thinking

Students should learn to use variables (letters) to represent mathematical quantities and expressions; they should be able to represent mathematical functions and relationships using tables, graphs, and equations. They should understand and correctly use positive and negative numbers, order of operations, formulas, equations, and inequalities. They should recognize the ways in which one quantity changes in relation to another.

Measurement

Students should learn the fundamental concepts of measurement through concrete experiences. They should be able to measure distance, mass (weight), time, capacity, temperature, and angles. They should learn to calculate simple perimeters, areas, and volumes. They should be able to perform measurement in both metric and customary systems using the appropriate tools and levels of precision.

Geometry

Students should understand the geo-

metric concepts necessary to function effectively in the three-dimensional world. They should have knowledge of such concepts as parallelism, perpendicularity, congruence, similarity, and symmetry. Students should know properties of simple plane and solid geometric figures. Students should visualize and verbalize how objects move in the world around them using such terms as slides, flips, and turns. Geometric concepts should be explored in settings that involve problem solving and measurement.

Statistics

Students should plan and carry out the collection and organization of data to answer questions in their everyday lives. Students should know how to construct, read, and draw conclusions from simple tables, maps, charts, and graphs. They should be able to present information about numerical data, such as measures of central tendency (mean, median, mode) and measures of dispersion (range, deviation). Students should recognize the basic uses and misuses of statistical representation and inference.

Probability

Students should understand elementary notions of probability to determine the likelihood of future events. They should identify situations in which immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make such predictions as election results, business forecasts, and outcomes of sporting events. They should learn how probability applies to research results and to the decision-making process.

Climate for Learning

To learn the essential mathematics needed for the twenty-first century, students need a nonthreatening environment in which they are encouraged to ask questions and take risks. The learning climate should incorporate high expectations for all students, regardless of sex, race, handicapping condition, or socioeconomic status. Students need to explore mathematics using manipulatives, measuring devices, models, calculators, and computers. They need to have opportunities to talk to each other about mathematics.

Students need modes of instruction that are suitable for the increased emphasis on problem solving, applications, and higher-order thinking skills. For example, cooperative learning allows students to work together in problem-solving situations to pose questions, analyze solutions, try alternative strategies, and check for reasonableness of results.

To implement the new instructional strategies, extensive professional development opportunities as well as new learning materials will be needed. ■